Exploring Teachers' Knowledge for Teaching Mathematics

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This paper reports some initial results from a research project[1] aimed at identifying and describing teaching approaches which are effective in improving numeracy learning in primary classrooms. To ascertain the influences on the teachers' intentions we posed a task that sought some indication of what aspects of a topic the teachers might consider when planning a set of experiences for their students. It seems that the teachers had a limited understanding of the ways that the concepts develop and the types of experiences that they might choose for their students. We argue that teacher education could focus more on key aspects of teaching the important topics of the mathematics curriculum.

Knowledge for Teaching

An old Latin cliché goes something like "if you don't know what harbour you are going to, then no wind is a good wind". We believe this applies to teaching mathematics. Assuming that the components of teaching mathematics include researching, planning, teaching (including explaining, interacting, listening), assessing, reporting and evaluating, it seems that each of these elements are informed by what the teacher knows about the subject matter. This knowledge includes the concepts themselves, as well as understandings about the way the concepts develop for students individually and collectively, and knowledge of the types of experiences that are likely to foster student learning.

We are focusing here on teacher knowledge rather than on teacher attitudes (see for example, Bell, Costello, & Kuchemann, 1983) or beliefs (see, for example, Perry, Howard, & Conroy, 1996). Clearly effective teaching is a mix of appropriate knowledge, attitudes and beliefs, but we argue that even with positive attitudes and beliefs, without the necessary knowledge it is unlikely that teachers can optimise the effectiveness of their teaching.

The importance of the knowledge and intention of the teacher is now widely acknowledged. Cobb and McClain (1999), for example, argued that teachers should have a clear impression of the direction that the learning of the individuals and the class will take. They proposed that the teacher should form an "instructional sequence (that) takes the form of a conjectured learning trajectory that culminates with the mathematical ideas that constitute our overall instructional intent" (p. 24). Likewise, the Early Numeracy Research

Project (ENRP) (see Clarke & Cheeseman, 2000) was based on a view that learning mathematics is about forming, linking and extending a few key ideas, with these key ideas providing the basis of planning, teaching and assessment. Teaching was seen as active, structured and explicit, while maximising the engagement of the students in carefully selected tasks and experiences. It is interesting to note that the case studies in that project identified teacher knowledge of both the content and the processes for teaching the content as key characteristics of effective teachers.

Teacher knowledge has been categorised into content knowledge, pedagogical content knowledge and pedagogical knowledge (Shulman, 1986). Lappan and Theule-Lubienski (1992) also proposed three key kinds of teacher knowledge: knowledge of mathematics, knowledge of students, and knowledge of the pedagogy of mathematics. Our project focuses mainly on pedagogical processes, and we are interested in gaining some insights into types of knowledge that influences the pedagogical processes teachers choose to use. We also consider that there are three phases for each teaching event: first is the preliminary phase of researching and planning, next is the actual teaching, and the third phase is assessing, reporting and evaluating. Each of these phases are informed by both teachers' content knowledge, and their pedagogical content knowledge.

Mewborn (2001) summarised and critiqued research of teacher content knowledge. She identified particular stands in that research including studies that sought to examine the relationship between teacher knowledge and student achievement, those that sought to characterise strengths and weaknesses in teachers' knowledge, those that compared the knowledge of different groups, such as primary and secondary teachers, and qualitative studies that sought to explicate relationships and to clarify the obvious complexity of the issues. Mewborn argued that, on balance it seemed that teachers' conceptions of mathematics did influence the way it is taught, but that while knowing the mathematics is important it is not the same as knowing how to teaching it. Mewborn called for research in a broader range of topics, for longitudinal research, and research into the conceptions of teachers who do have a firm understanding of the discipline.

Pedagogical content knowledge, according to Shulman (1986), is "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9). It includes knowledge of the most useful forms of representation, the most useful metaphors, and the most powerful ways of explaining or illustrating particular aspects of the content domain to make it comprehensible to others. It also includes an understanding of what makes the learning of specific topics easy or difficult and the common misconceptions and likely preconceptions that students may bring to the learning of specific content areas.

Ma (1999) captures the importance and interdependence of these two forms of knowledge powerfully in her comparative study of Chinese and American teachers of mathematics. Ma found that Chinese teachers were more likely to have a "profound understanding of fundamental mathematics" (p. xxiii) than their American counterparts and that this difference parallelled the comparative difference in student achievement. This understanding included not only a deeper knowledge of subject matter but also a knowledge of *how* to teach it – Ma refers to this as "*teachers*' subject matter knowledge" (p.145, Ma's emphasis).

This subject matter knowledge does not only refer to lists of strategies for teaching mathematics (e.g. Sullivan, 1999), or to the topics that are commonly prominent in mathematics teacher education, both pre- and in-service, such as co-operative groups, using concrete materials, problem solving strategies, and metacognition. More important is

whether teachers can delineate the key phases through which students' learning develops, the types of experiences that are necessary at those phases, and the ways such experiences can be differentiated for particular students.

Seeking Insights into Teachers' Knowledge for Teaching

Clearly, there are methodological and ethical difficulties in researching teacher knowledge that are exacerbated in the case of mathematics teaching by lack of access to a well-developed, meta-language to talk about the teaching of mathematics. To overcome these, and with a view to providing potential resources for teacher professional development, an instrument was developed that would allow participant involvement in a non-threatening and professionally respectful way.

All teachers of mathematics at the 16 research schools[2] were asked to complete the task at the first research schools meeting held in October, 2001. Teachers working at similar levels, P-2, 3-4 or 5-6, were asked to work in threes to prepare a concept map for a particular topic. Teachers were also asked to list the key pre-requisite ideas, describe an illustrative teaching task, indicate connections to other key learning areas, and suggest an open-ended question or investigation, an assessment task, and a real world or practical activity. This exercise was located in the context of a joint planning activity and supported by a common format.

Year 5-6 teachers considered the teaching of place value, Year 3-4 teachers considered multiplication, and Year P-2 teachers looked at teaching length. Coded scoring rubrics were developed for each aspect of the task where 0 represented the lowest rating and 3 the highest. Most scores in the rubrics refer to two aspects: usually one relates to the legitimacy of the particular example; and the other refers to the adequacy of the elements of the concept being considered. Both were evaluated by reference to research relevant to the particular concept considered.

We focus here on only four of these components. The instructions for these components were for the teachers to record:

- A concept map for the set of activities. This should contain, at least, identification of the key experiences or concepts associated with the particular topics, and lines to illustrate connections. Use as few words as possible.
- The key ideas that the teacher should be thinking about.
- An illustrative task or activity
- An open-ended questions or investigation that could be used

We based our scoring of the concept of length on the framework developed by the ENRP. The key elements of the framework for length are comparing (including conservation and transitivity), quantifying (including the idea of a unit iteration), using standard units, and applying (see Sullivan & McDonough, 2002 for elaboration of this). The teachers completed the instrument in groups of three, all teaching at the same level, but not necessarily from the same school. The responses were scored by two members of the project team, in collaboration, and then scored again by another member of the team. Where there were discrepancies, these were checked and debated. Basically we expected that the teachers, in their response, would indicated their understanding of the particular strategy and of the key elements in the learning of length. For example, the ENRP noted

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[2] A structured, representative sample of Victorian primary schools including 4 Catholic schools, I Independent school and 1 special school.

that three quarters of students at the start of schooling can already compare the length of two objects. While we did not expect the teachers to be aware of this result, we assumed they should be aware what their students can do. So tasks at Grade 1 level that focus on simple comparisons, for example, received a low rating.

The percentage of responses assigned a particular score for the concept mapping task for length is shown in Table 1. Note that concept maps have been used elsewhere as a way of accessing teacher conceptual understanding in mathematics (e.g., Williams 1998), and this particular prompt was the focus of the task.

Table 1

Percentage Ratings for Teaching Length in Year 1 in Concept Maps (N=83)

Rating	Rating descriptor	Percent
0	Lines not illustrative of conceptual connections	57
1	Some lines illustrative of connections, others not	41
2	For concepts shown, lines are relevant, indicative of connections	2
3	Lines illustrative of links and sequences, clearly shown	0

Even though the teachers worked in groups (or perhaps because of that) the concepts maps did not seem to indicate a firm understanding of the links and connections associated with the teaching of length.

Figure 1 presents an example of a concept map that was rated as "lines not illustrative of conceptual connections":
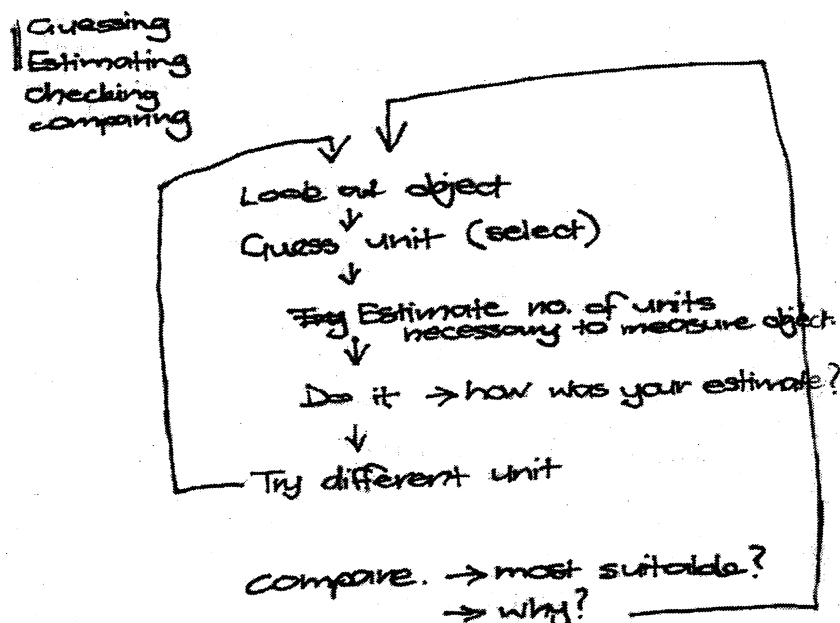

Figure 1. A concept map "not illustrative of conceptual connections".

While there are some relevant elements present in the map, there is no real sense of connections and logical links. To the extent that this map is a representation of how these teachers see the concept of length, it is difficult to anticipate that their teaching of this topic would be focused or coherent.

We also sought responses to prompts that would provide different insights into their perspectives on the teaching of length. Table 2 presents the scoring of responses to the prompt that asked for the key pre-requisite/subsequent ideas for length to be listed.

Table 2

Percentage Ratings for the Key Ideas for Teaching Length In Year 1 (N=83)

Rating	Rating descriptor	Percent
0	Lists ideas, no clear links	45
1	Some prior and subsequent ideas	43
2	Most key prior and subsequent ideas	10
3	Key prior and subsequent ideas	2

Most responses were scored in the lowest two categories. Again it seems that the teachers overall have not articulated an understanding of the way these key ideas develop. To illustrate the way the responses were interpreted, the following is an example rated as "lists ideas, no clear links":

> Discover children's prior knowledge of language in relation to length

It is possible that this group saw prior knowledge as a key idea. However, given the context of the prompt, we required the teachers to incorporate some the elements of learning length into their responses. Generally the responses at this level were similarly lacking in detail.

The following is an example that was rated at "some prior and subsequent ideas":

> Prior: knowledge of long/short/same as. Placing objects end to end. Ordering of length of objects

In this case, some of the relevant elements are present, although it is clearly only a limited subset of the possible concepts in length at this level. This response is representative of the other responses scored at this level. We expected a broader range of the key ideas to be presented. Note that nearly 90% of the teachers' responses were rated at or below this level.

The following is an example that was rated at "most key prior and subsequent ideas":

> Basic vocabulary of length e.g., long and short. Things can be measured using a variety of equipment – formal/informal. The need for appropriate units of measure – using relevant equipment

While this level of description was presented by comparatively few teachers, we can infer that these teachers, at least as a group, could plan effective experiences for teaching length.

The following is an example that was rated at "key prior and subsequent ideas":

> Concepts of long and short. Length and width. Informal tools to measure (making appropriate choices). Estimating and comparing lengths. Need for a standard unit of measure

These teachers have touched the relevant bases, and the response is indicative of a firm understanding of the key elements of the teaching of length.

It is possible that teachers know more than they recorded on the response sheets. However there results are still cause for concern.

A further perspective was sought through the teachers' suggestions of an illustrative teaching task. The rating of responses is presented in Table 3.

Table 3

Percentage Ratings for the Illustrative Task for Teaching Length in Year 1 (N=83)

Rating	Rating descriptor	Percent
0	Focus or purpose of task not clear	20
1	Task emphasizes skill or application only	45
2	Useful task but loosely connected to concept	33
3	Task emphasizes concept or its understanding	2

The responses received higher ratings for this task. An example of an illustrative task that was rated at "focus or purpose of task not clear" was:

> Make a paper chain the length of your ruler using common units

While the task may seem reasonable, it was scored at the lowest level because it seems a mix between a chain with presumably variable loops and some other task. Either it does not make sense or it is trivial.

An example of an illustrative task that was rated as "task emphasizes skill or application only" was:

> Handspans- create cut paste student handspans. Establish the need for a common language to share information about distance.

While at least in this case, the units for an individual would be the same, it is not clear what the students would be required to measure.

An example of an illustrative task that was rated as "useful task, but loosely connected to concept" was:

> Who has the longest name. Graph results.

One way, this is just a counting and graphing task. There is though considerable potential in the task and so it was scored more highly than the previous example.

An example of an illustrative task activity that was rated as "task emphasizes concept or its understanding" was:

> Bring your soft toy from home. Measure its length ("how long it is") using a strip of paper. Find another toy that's the same length.

Even though it is a low level and easy task at this level, it does have some openness and some possibility for exploring problematic aspects of measuring the length of toys.

Overall the teachers seemed slightly better at specifying illustrative tasks, although it should be noted that this does not provide evidence of their ability to plan sets of coherent experiences.

We also asked the teachers to give an example of a relevant open-ended question. The profile of the ratings of their responses is presented in Table 4.

Table 4

Percentage Ratings for the Open-Ended Questions for Teaching Length in Year 1 (N=83)

Rating	Rating descriptor	Percent
0	Closed or inappropriate activity	26
1	Some openness but only loosely linked to the concept	67
2	Open and loosely linked to one of the key ideas	5
3	Open, illustrates or elaborates key ideas	2

An example of a response that was rated as "closed or inappropriate activity" was:

> How do we measure? Why do we measure?

This response was scored at the lowest level because it is inappropriate in that it does not necessarily focus on length, and in any case it is hard to know what the students would be expected to say.

An example of an activity that was rated as "some openness but only loosely linked to the concept" was:

> Measure your own arm, leg etc. Find someone else in the room with the same measurements. Measure other body parts. Find total length of bones in your body

This task has some potential but it was written in a way that does not convey an appreciation of the type of investigation needed by students at this level.

An example of an activity that was rated as "open and loosely linked to one of the key ideas" was:

> Find things in the room that are longer than your hand but shorter than your arm

This is both clear and open in that it prompts and allows a variety of responses. It does, though, focus on just the comparing aspect of length.

An example of an activity that was open, and illustrated or elaborated key idea was:

> What objects that you can see in the classroom could you use to measure the netball court; picture of the blue whale; the oval?

This also allows a variety of response, and has the advantage that it addresses a key aspects of measuring for students at this level, that of appropriate units of measure.

Overall, the responses to the concept mapping, the key ideas, and the open-ended question prompts were sufficiently low to be of concern. The responses to the illustrative task were a little better. While there are a number of reasons to suspect that these responses under-represent the actual pedagogical content knowledge of the teachers, they do indicate some potential concerns about the depth of teachers' knowledge of key aspects of teaching length, and subject to further exploration, may suggest some potential alternate emphases in mathematics teacher education.

Conclusion

Teachers' knowledge is recognised as a key component of their capacity to plan, implement and evaluate mathematics learning experiences. *The Primary Numeracy Research Project* is concerned with identifying and describing a range of numeracy-specific teaching approaches. To the extent that teachers' knowledge shapes their intentions and subsequent actions in the classroom, the project is interested in exploring its

nature and role in planning and implementing classroom interactions aimed at scaffolding student numeracy learning.

The evidence presented suggests that these teachers did not include in their responses the type of knowledge of the teaching of length that would make us confident in the adequacy of their conceptualisation of the key elements of its teaching and learning. The responses of the middle primary teachers on multiplication, and of the upper primary teaching on decimal place value were scored lower.

It is possible that the context of the data collection did not allow the teachers to give full responses. They were working in groups, possibly including some unfamiliar colleagues. They were perhaps satisfied with a general response when they may have produced more details had they actually been planning classroom experiences. Nevertheless, that they chose these particular responses rather than others is possibly evidence of a lack of awareness of the nature of the learning of length.

This was a preliminary instrument and was developed more as the basis of some subsequent professional development. Nevertheless, taken on face value, the data raise concerns about the level of teachers' pedagogical content knowledge. We suspect that it may mean that teacher education programs should focus more on key aspects of teaching and learning the important concepts in mathematics.

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